Chapter 1

Logical Validity and Soundness

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This short paper reproduces the first chapter of a text, Symbolic Logic by me. The complete text is available online at http://rocket.csusb.edu/~troy/int-ml.html. In the chapter, I introduce two central notions for argument evaluation. The presentation is completely informal. It is possible to develop formal methods for working with validity and soundness, but it is also possible to apply the informal notions directly to problems in philosophy and beyond. In either case, it is important to understand the basic notions, in order to understand what is accomplished in reasoning. Exercises are included, with answers to selected exercises at the end.

Symbolic logic is a tool or machine for the identification of argument goodness. It makes sense to begin, however, not with the machine, but by saying something about this argument goodness that the machinery is supposed to identify. That is the task of this chapter.

But first, we need to say what an argument is. An argument is made up of sentences one of which is taken to be supported by the others.

AR An argument is some sentences, one of which (the conclusion) is taken to be supported by the remaining sentences (the premises).

(Important definitions are often offset and given a short name as above; then there may be appeal to the definition by its name, in this case, ‘AR’.) So an argument has premises which are taken to support a conclusion. Such support is often indicated by words or phrases of the sort, ‘so’, ‘it follows’, ‘therefore’, or the like. We will typically indicate the division by a simple line between premises and conclusion. Roughly, an
argument is good if the premises do what they are taken to do, if they actually support the conclusion. An argument is bad if they do not accomplish what they are taken to do, if they do not actually support the conclusion.

Logical validity and soundness correspond to different ways an argument can go wrong. Consider the following two arguments:

(A)  
Only citizens can vote  
Hannah is a citizen  
Hannah can vote

(B)  
All citizens can vote  
Hannah is a citizen  
Hannah can vote

The line divides premises from conclusion, indicating that the premises are supposed to support the conclusion. Thus these are arguments. But these arguments go wrong in different ways. The premises of argument (A) are true; as a matter of fact, only citizens can vote, and Hannah (my daughter) is a citizen. But she cannot vote; she is not old enough. So the conclusion is false. Thus, in argument (A), the relation between the premises and the conclusion is defective. Even though the premises are true, there is no guarantee that the conclusion is true as well. We will say that this argument is *logically invalid*. In contrast, argument (B) is logically valid. If its premises were true, the conclusion would be true as well. So the relation between the premises and conclusion is not defective. The problem with this argument is that the premises are not true — not all citizens can vote. So argument (B) is defective, but in a different way. We will say that it is *logically unsound*.

The task of this chapter is to define and explain these notions of logical validity and soundness. I begin with some preliminary notions, then turn to official definitions of logical validity and soundness, and finally to some consequences of the definitions.

### 1.1 Consistent Stories

Given a certain notion of a *possible or consistent* story, it is easy to state definitions for logical validity and soundness. So I begin by identifying the kind of stories that matter. Then we will be in a position to state the definitions, and apply them in some simple cases.

Let us begin with the observation that there are different sorts of possibility. Consider, say, “Hannah could make it in the WNBA.” This seems true. She is reasonably athletic, and if she were to devote herself to basketball over the next few years, she might very well make it in the WNBA. But wait! Hannah is only a kid — she rarely gets the ball even to the rim from the top of the key — so there is no way she could make it in the WNBA. So she both could and could not make it. But this
cannot be right! What is going on? Here is a plausible explanation: Different sorts of possibility are involved. When we hold fixed current abilities, we are inclined to say there is no way she could make it. When we hold fixed only general physical characteristics, and allow for development, it is natural to say that she might. The scope of what is possible varies with whatever constraints are in play. The weaker the constraints, the broader the range of what is possible.

The sort of possibility we are interested in is very broad, and constraints are correspondingly weak. We will allow that a story is possible or consistent so long as it involves no internal contradiction. A story is impossible when it collapses from within. For this it may help to think about the way you respond to ordinary fiction. Consider, say, J.K. Rowling’s *Harry Potter and the Prisoner of Azkaban* (much loved by my youngest daughter). Harry and his friend Hermione are at wizarding school. Hermione acquires a “time turner” which allows time travel, and uses it in order to take classes that are offered at the same time. Such devices are no part of the actual world, but they fit into the wizarding world of Harry Potter. So far, then, the story does not contradict itself. So you go along.

At one stage, though, Harry is at a lakeshore under attack by a bunch of fearsome “dementors.” His attempts to save himself appear to have failed when a figure across the lake drives the dementors away. But the figure who saves Harry is Harry himself who has come back from the future. Somehow, then, as often happens in these stories, the past depends on the future, at the same time as the future depends on the past: Harry is saved only insofar as he comes back from the future, but he comes back from the future only insofar as he is saved. This, rather than the time travel itself, generates an internal conflict. The story makes it the case that you cannot have Harry’s rescue apart from his return, and cannot have Harry’s return apart from his rescue. This might make sense if time were always repeating in an eternal loop. But, according to the story, there were times before the rescue and after the return. So the story faces internal collapse. Notice: the objection does not have anything to do with the way things actually are — with existence of time turners or the like; it has rather to do with the way the story hangs together internally.\(^1\) Similarly, we want to ask whether stories hold together internally. If a story holds together internally, it counts for our purposes

\(^1\)In more consistent cases of time travel (in fiction) time seems to move on different paths so that after yesterday and today, there is another yesterday and another today. So time does not return to the very point at which it first turns back. In the trouble cases, time seems to move in a sort of “loop” so that a point on the path to today (this very day) goes through tomorrow. With this in mind, it is interesting to think about say, the *Terminator* and *Back to the Future* movies along with, maybe more consistent, Hermione’s “academic” travel or *Groundhog Day*. Even if I am wrong, and the Potter story is internally consistent, the overall point should be clear. And it should be clear that I am not saying anything serious about time travel.
as consistent and possible. If a story does not hold together, it is not consistent or possible.

In some cases, stories may be consistent with things we know are true in the real world. Thus Hannah could grow up to play in the WNBA. There is nothing about our world that rules this out. But stories may remain consistent though they do not fit with what we know to be true in the real world. Here are cases of time travel and the like. Stories become inconsistent when they collapse internally — as when a story says that some time both can and cannot happen apart from another.

As with a movie or novel, we can say that different things are true or false in our stories. In *Harry Potter* it is true that Harry and Hermione travel through time with a timer turner, but false that they go through time in a DeLorean (as in the *Back to the Future* films). In the real world, of course, it is false that there are time turners, and false that DeLoreans go through time. Officially, a complete story is always maximal in the sense that any sentence is either true or false in it. A story is inconsistent when it makes some sentence both true and false. Since, ordinarily, we do not describe every detail of what is true and what is false when we tell a story, what we tell is only part of a maximal story. In practice, however, it will be sufficient for us merely to give or fill in whatever details are relevant in a particular context.

But there are a couple of cases where we cannot say when sentences are true or false in a story. The first is when stories we tell do not fill in relevant details. In *The Wizard of Oz*, it is true that Dorothy wears red shoes. But neither the movie nor the book have anything to say about whether she likes Twinkies. By themselves, then, neither the book nor the movie give us enough information to tell whether “Dorothy likes Twinkies” is true or false in the story. Similarly, there is a problem when stories are inconsistent. Suppose according to some story,

(a) All dogs can fly
(b) Fido is a dog
(c) Fido cannot fly

Given (a), all dogs fly; but from (b) and (c), it seems that not all dogs fly. Given (b), Fido is a dog; but from (a) and (c) it seems that Fido is not a dog. Given (c), Fido cannot fly; but from (a) and (b) it seems that Fido can fly. The problem is not that inconsistent stories say too little, but rather that they say too much. When a story is inconsistent, we will refuse to say that it makes any sentence (simply) true or false.\(^2\)

\(^2\)The intuitive picture developed above should be sufficient for our purposes. However, we are on the verge of vexed issues. For further discussion, you may want to check out the vast literature on “possible worlds.” Contributions of my own include the introductory article, “Modality,” in *The Continuum Companion to Metaphysics*. 
It will be be helpful to consider some examples of consistent and inconsistent stories:

(a) The real story, “Everything is as it actually is.” Since no contradiction is actually true, this story involves no contradiction; so it is internally consistent and possible.

(b) “All dogs can fly: over the years, dogs have developed extraordinarily large and muscular ears; with these ears, dogs can fly.” It is bizarre, but not obviously inconsistent. If we allow the consistency of stories according to which monkeys fly, as in *The Wizard of Oz*, or elephants fly, as in *Dumbo*, then we should allow that this story is consistent as well.

(c) “All dogs can fly, but my dog Fido cannot; Fido’s ear was injured while he was chasing a helicopter, and he cannot fly.” This is *not* internally consistent. If all dogs can fly and Fido is a dog, then Fido can fly. You might think that Fido retains a sort of flying nature — just because Fido remains a dog. In evaluating internal consistency, however, we require that *meanings remain the same*.

(C) 

\[
\begin{array}{c|c|c}
\text{able to fly} & \text{All dogs can fly} & \text{T} \\
\hline
\text{T} & \text{Fido is a dog} & \text{F} \\
\hline
\text{F} & \text{Fido cannot fly} & \text{T} \\
\end{array}
\]

If “can fly” means “is able to fly” then in the story it is true that Fido cannot fly, but not true that all dogs can fly (since Fido cannot). If “can fly” means “has a flying nature” then in the story it is true that all dogs can fly, but not true that Fido cannot (because he remains a dog). The only way to keep both ‘all dogs fly’ and ‘Fido cannot fly’ true is to *switch* the sense of “can fly” from one use to another. So long as “can fly” means the same in each use, the story is sure to fall apart insofar as it says both that Fido is and is not that sort of thing.

(d) “Germany won WWII; the United States never entered the war; after a long and gallant struggle, England and the rest of Europe surrendered.” It did not happen; but the story does not contradict itself. For our purposes, then it counts as possible.

(e) “1 + 1 = 3; the numerals ‘2’ and ‘3’ are switched (the numerals are ‘1’, ‘3’, ‘2’, ‘4’, ‘5’, ‘6’, ‘7’…); so that one and one are three.” This story does not hang together. Of course numerals can be switched — so that people would correctly say, ‘1 + 1 = 3’. But this does not make it the case that one and one are three! We tell stories in our own language (imagine that you are describing a foreign-language film in English). Take a language like English except that ‘fly’ means ‘bark’; and consider a movie where dogs are ordinary, so that people in the movie correctly assert, in their language, ‘dogs fly’. But changing the words people use to describe a situation does
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not change the situation. It would be a mistake to tell a friend, in English, that you saw a movie in which there were flying dogs. Similarly, according to our story, people correctly assert, in their language, ‘1 + 1 = 3’. But it is a mistake to say in English (as our story does), that this makes one and one equal to three.

Some authors prefer talk of “possible worlds,” “possible situations” or the like to that of consistent stories. It is conceptually simpler to stick with stories, as I have, than to have situations and distinct descriptions of them. However, it is worth recognizing that our consistent stories are or describe possible situations, so that the one notion matches up directly with the others.

As you approach the following exercises, note that answers to problems indicated by star are provided in the back of the book. It is essential to success that you work a significant body of exercises successfully and independently. So do not neglect exercises!

E1.1. Say whether each of the following stories is internally consistent or inconsistent. In either case, explain why.

*a. Smoking cigarettes greatly increases the risk of lung cancer, although most people who smoke cigarettes do not get lung cancer.

b. Joe is taller than Mary, but Mary is taller than Joe.

*c. Abortion is always morally wrong, though abortion is morally right in order to save a woman’s life.

d. Mildred is Dr. Saunders’s daughter, although Dr. Saunders is not Mildred’s father.

*e. No rabbits are nearsighted, though some rabbits wear glasses.

f. Ray got an ‘A’ on the final exam in both Phil 200 and Phil 192. But he got a ‘C’ on the final exam in Phil 192.

*g. Barack Obama was never president of the United States, although Michelle is president right now.

h. Egypt, with about 100 million people is the most populous country in Africa, and Africa contains the most populous country in the world. But the United States has over 200 million people.
i. The death star is a weapon more powerful than that in any galaxy, though there is, in a galaxy far, far away, a weapon more powerful than it.

j. Luke and the Rebellion valiantly battled the evil Empire, only to be defeated. The story ends there.

E1.2. For each of the following sentences, (i) say whether it is true or false in the real world and then (ii) say, if you can, whether it is true or false according to the accompanying story. In each case, explain your answers. Do not forget about contexts where we refuse to say whether sentences are true or false. The first problem is worked as an example.

a. Sentence: Aaron Burr was never a president of the United States.
   Story: Aaron Burr was the first president of the United States, however he turned traitor and was impeached and then executed.
   (i) It is true in the real world that Aaron Burr was never a president of the United States. (ii) But the story makes the sentence false, since the story says Burr was the first president.

b. Sentence: In 2006, there were still buffalo.

*c. Sentence: After overrunning Phoenix in early 2006, a herd of buffalo overran Newark, New Jersey.

d. Sentence: There has been an all-out nuclear war.
   Story: After the all-out nuclear war, John Connor organized the Resistance against the machines — who had taken over the world for themselves.

*e. Sentence: Jack Nicholson has swum the Atlantic.
   Story: No human being has swum the Atlantic. Jack Nicholson and Bill Clinton and you are all human beings, and at least one of you swam all the way across!
f. Sentence: Some people have died as a result of nuclear explosions.
   Story: As a result of a nuclear blast that wiped out most of this continent, you
   have been dead for over a year.

*g. Sentence: Your instructor is not a human being.
   Story: No beings from other planets have ever made it to this country. However,
   your instructor made it to this country from another planet.

h. Sentence: Lassie is both a television and movie star.
   Story: Dogs have super-big ears and have learned to fly. Indeed, all dogs can
   fly. Among the many dogs are Lassie and Rin Tin Tin.

*i. Sentence: The Yugo is the most expensive car in the world.
   Story: Jaguar and Rolls Royce are expensive cars. But the Yugo is more
   expensive than either of them.

j. Sentence: Lassie is a bird who has learned to fly.
   Story: Dogs have super-big ears and have learned to fly. Indeed, all dogs can
   fly. Among the many dogs are Lassie and Rin Tin Tin.

1.2 The Definitions

The definition of logical validity depends on what is true and false in consistent stories. The definition of soundness builds directly on the definition of validity. Note: in offering these definitions, I stipulate the way the terms are to be used; there is no attempt to say how they are used in ordinary conversation; rather, we say what they will mean for us in this context.

LV An argument is logically valid if and only if (iff) there is no consistent story in
   which all the premises are true and the conclusion is false.

LS An argument is logically sound iff it is logically valid and all of its premises are
   true in the real world.

Observe that logical validity has entirely to do with what is true and false in consistent stories. Only with logical soundness is validity combined with premises true in the real world.

Logical (deductive) validity and soundness are to be distinguished from inductive
validity and soundness or success. For the inductive case, it is natural to focus on the
plausibility or the probability of stories — where an argument is relatively strong when stories that make the premises true and conclusion false are relatively implausible. Logical (deductive) validity and soundness are thus a sort of limiting case, where stories that make premises true and conclusion false are not merely implausible, but impossible. In a deductive argument, conclusions are supposed to be guaranteed; in an inductive argument, conclusions are merely supposed to be made probable or plausible. For mathematical logic, we set the inductive case to the side, and focus on the deductive.

Also, do not confuse truth with validity and soundness. A sentence is true in the real world when it correctly represents how things are in the real world, and true in a story when it correctly represents how things are in the story. An argument is valid when there is no consistent story that makes the premises true and conclusion false, and sound when it is valid and all its premises are true in the real world. The definitions for validity and soundness depend on truth and falsity for the premises and conclusion in stories and then in the real world. But, just as it would be a mistake to say that the number three weighs eleven pounds, so truth and falsity do not even apply to arguments themselves, which may be valid or sound.3

1.2.1 Invalidity

It will be easiest to begin thinking about invalidity. From the definition, if an argument is logically valid, there is no consistent story that makes the premises true and conclusion false. So to show that an argument is invalid, it is enough to produce even one consistent story that makes premises true and conclusion false. Perhaps there are stories that result in other combinations of true and false for the premises and conclusion; this does not matter for the definition. However, if there is even one story that makes premises true and conclusion false then, by definition, the argument is not logically valid — and if it is not valid, by definition, it is not logically sound. We can work through this reasoning by means of a simple invalidity test. Given an argument, this test has the following four stages.

IT  a. List the premises and negation of the conclusion.
   b. Produce a consistent story in which the statements from (a) are all true.
   c. Apply the definition of validity.
   d. Apply the definition of soundness.

3From an introduction to philosophy of language, one might wonder (with good reason) whether the proper bearers of truth are sentences rather than, say, propositions. This question is not relevant to the simple point made above.
We begin by considering what needs to be done to show invalidity. Then we do it. Finally we apply the definitions to get the results. For a simple example, consider the following argument,

\[(D)\]

Eating brussels sprouts results in good health

Ophelia has good health

Ophelia has been eating brussels sprouts

The definition of validity has to do with whether there are consistent stories in which the premises are true and the conclusion false. Thus, in the first stage, we simply write down what would be the case in a story of this sort.

a. List premises and negation of conclusion.

In any story with the premises true and conclusion false,

1. Eating brussels sprouts results in good health
2. Ophelia has good health
3. Ophelia has not been eating brussels sprouts

Observe that the conclusion is reversed! At this stage we are not giving an argument. Rather we merely list what is the case when the premises are true and conclusion false. Thus there is no line between premises and the last sentence, insofar as there is no suggestion of support. It is easy enough to repeat the premises for (1) and (2). Then for (3) we say what is required for the conclusion to be false. Thus, “Ophelia has been eating brussels sprouts” is false if Ophelia has not been eating brussels sprouts. I return to this point below, but that is enough for now.

An argument is invalid if there is even one consistent story that makes the premises true and the conclusion false. Thus, to show invalidity, it is enough to produce a consistent story that makes the premises true and conclusion false.

b. Produce a consistent story in which the statements from (a) are all true.

Story: Eating brussels sprouts results in good health, but eating spinach does so as well; Ophelia is in good health but has been eating spinach, not brussels sprouts.

For the statements listed in (a): the story satisfies (1) insofar as eating brussels sprouts results in good health; (2) is satisfied since Ophelia is in good health; and (3) is satisfied since Ophelia has not been eating brussels sprouts. The story explains how she manages to maintain her health without eating brussels sprouts, and so the consistency of (1) - (3) together. The story does not have to be true — and, of course,
many different stories will do. All that matters is that there is a consistent story in which the premises of the original argument are true, and the conclusion is false.

Producing a story that makes the premises true and conclusion false is the creative part. What remains is to apply the definitions of validity and soundness. By LV, an argument is logically valid only if there is no consistent story in which the premises are true and the conclusion is false. So if, as we have demonstrated, there is such a story, the argument cannot be logically valid.

c. Apply the definition of validity. This is a consistent story that makes the premises true and the conclusion false; thus, by definition, the argument is not logically valid.

By LS, for an argument to be sound, it must have its premises true in the real world and be logically valid. Thus if an argument fails to be logically valid, it automatically fails to be logically sound.

d. Apply the definition of soundness. Since the argument is not logically valid, by definition, it is not logically sound.

Given an argument, the definition of validity depends on stories that make the premises true and the conclusion false. Thus, in step (a) we simply list claims required of any such story. To show invalidity, in step (b), we produce a consistent story that satisfies each of those claims. Then in steps (c) and (d) we apply the definitions to get the final results; for invalidity, these last steps are the same in every case.

It may be helpful to think of stories as a sort of “wedge” to pry the premises of an argument off its conclusion. We pry the premises off the conclusion if there is a consistent way to make the premises true and the conclusion not. If it is possible to insert such a wedge between the premises and conclusion, then a defect is exposed in the way premises are connected to the conclusion. Observe that the flexibility we allow in consistent stories (with flying dogs and the like) corresponds directly to the strength of the required connection between premises and conclusion. If the connection is sufficient to resist all such attempts to wedge the premises off the conclusion, then it is significant indeed. Observe also that our method reflects what we did with argument (A) at the beginning of the chapter: Faced with the premises that only citizens can vote and Hannah is a citizen, it was natural to worry that she might be under-age and so cannot vote. But this is precisely to produce a story that makes the premises true and conclusion false. Thus our method is not “strange” or “foreign”! Rather, it makes rigorous what has seemed natural from the start.

Here is another example of our method. Though the argument may seem on its face not to be a very good one, we can expose its failure by our methods — in fact,
again, our method may formalize or make rigorous a way you very naturally think about cases of this sort. Here is the argument,

(E) \[
\begin{align*}
&\text{I shall run for president} \\
&\text{I shall be one of the most powerful men on earth}
\end{align*}
\]

To show that the argument is invalid, we turn to our standard procedure.

a. In any story with the premise true and conclusion false,
   1. I shall run for president
   2. I shall not be one of the most powerful men on earth

b. Story: I do run for president, but get no financing and gain no votes; I lose the election. In the process, I lose my job as a professor and end up begging for scraps outside a Domino’s Pizza restaurant. I fail to become one of the most powerful men on earth.

c. This is a consistent story that makes the premise true and the conclusion false; thus, by definition, the argument is not logically valid.

d. Since the argument is not logically valid, by definition, it is not logically sound.

This story forces a wedge between the premise and the conclusion. Thus we use the definition of validity to explain why the conclusion does not properly follow from the premises. It is, perhaps, obvious that running for president is not enough to make me one of the most powerful men on earth. Our method forces us to be very explicit about why: running for president leaves open the option of losing, so that the premise does not force the conclusion. Once you get used to it, then, our method may appear as a natural approach to arguments.

If you follow this method for showing invalidity, the place where you are most likely to go wrong is stage (b), telling stories where the premises are true and the conclusion false. Be sure that your story is consistent, and that it verifies each of the claims from stage (a). If you do this, you will be fine.

E1.3. Use our invalidity test to show that each of the following arguments is not logically valid, and so not logically sound. Understand terms in their most natural sense.

*a. If Joe works hard, then he will get an ‘A’
   Joe will get an ‘A’
   Joe works hard
b. Harry had his heart ripped out by a government agent
   _____________
   Harry is dead

c. Everyone who loves logic is happy
   Jane does not love logic
   _____________
   Jane is not happy

d. Our car will not run unless it has gasoline
   Our car has gasoline
   _____________
   Our car will run

e. Only citizens can vote
   Hannah is a citizen
   _____________
   Hannah can vote

1.2.2 Validity

Suppose I assert that no student at California State University San Bernardino is from Beverly Hills, and attempt to prove it by standing in front of the library and buttonholing students to ask if they are from Beverly Hills — I do this for a week and never find anyone from Beverly Hills. Is the claim that no CSUSB student is from Beverly Hills thereby proved? Of course not — for there may be students I never met. Similarly, failure to find a story to make the premises true and conclusion false does not show that there is not one — for all we know, there might be some story we have not thought of yet. So, to show validity, we need another approach. If we could show that every story which makes the premises true and conclusion false is inconsistent, then we could be sure that no consistent story makes the premises true and conclusion false — and so, from the definition of validity, we could conclude that the argument is valid. Again, we can work through this by means of a procedure, this time a validity test.

VT
   a. List the premises and negation of the conclusion.
   b. Expose the inconsistency of such a story.
   c. Apply the definition of validity.
   d. Apply the definition of soundness.

In this case, we begin in just the same way. The key difference arises at stage (b). For an example, consider this argument.
Since LV has to do with stories where the premises are true and the conclusion false, as before, we begin by listing the premises together with the negation of the conclusion.

1. No car is a person
2. My mother is a person
3. My mother is not a car

Any story where “My mother is not a car” is false, is one where my mother is a car (perhaps along the lines of the 1965 TV series, My Mother the Car).

For invalidity, we would produce a consistent story in which (1) - (3) are all true. In this case, to show that the argument is valid, we show that this cannot be done. That is, we show that no story that makes each of (1) - (3) true is a consistent story.

The reasoning should be clear if you focus just on the specified lines. Given (1) and (3), if no car is a person and my mother is a car, then my mother is not a person. But then my mother is a person from (2) and not a person from (4). So we have our goal: any story with (1) - (3) as members contradicts itself and therefore is not consistent. Observe that we could have reached this result in other ways. For example, we might have reasoned from (1) and (2) that (4'), my mother is not a car; and then from (3) and (4') to the result that (5') my mother is and is not a car. Either way, an inconsistency is exposed. Thus, as before, there are different options for this creative part.

Now we are ready to apply the definitions of logical validity and soundness. First,
For the invalidity test, we produce a consistent story that “hits the target” from stage (a), to show that the argument is invalid. For the validity test, we show that any attempt to hit the target from stage (a) must collapse into inconsistency: no consistent story includes each of the elements from stage (a) so that there is no consistent story in which the premises are true and the conclusion false. So by application of LV the argument is logically valid.

Given that the argument is logically valid, LS makes logical soundness depend on whether the premises are true in the real world. Suppose we think the premises of our argument are in fact true. Then,

d. Apply the definition of soundness. In the real world no car is a person and my mother is a person, so all the premises are true; so since the argument is also logically valid, by definition, it is logically sound.

Observe that LS requires for logical soundness that an argument is logically valid and that its premises are true in the real world. Thus we are no longer thinking about merely possible stories! Soundness depends on the way things are in the real world. And we do not say anything at this stage about claims other than the premises of the original argument! Thus we do not make any claim about the truth or falsity of the conclusion, “my mother is not a car.” Rather, the observations have entirely to do with the two premises, “no car is a person” and “my mother is a person.” When an argument is valid and the premises are true in the real world, by LS, it is logically sound.

But it will not always be the case that a valid argument has true premises. Say My Mother the Car is (surprisingly) a documentary about a person reincarnated as a car (the premise of the show) and therefore a true account of some car that is a person. Then some cars are persons and the first premise is false; so you would have to respond as follows,

d’. Since in the real world some cars are persons, the first premise is not true. So, though the argument is logically valid, by definition it is not logically sound.

Another option is that you are in doubt about reincarnation into cars, and in particular about whether some cars are persons. In this case you might respond as follows,

d”. Although in the real world my mother is a person, I cannot say whether no car is a person; so I cannot say whether the first premise is true. So though the argument is logically valid, I cannot say whether it is logically sound.

So once we decide that an argument is valid, for soundness there are three options:
(i) You are in a position to identify all of the premises as true in the real world. In this case, you should do so, and apply the definition for the result that the argument is logically sound.

(ii) You are in a position to say that at least one of the premises is false in the real world. In this case, you should do so, and apply the definition for the result that the argument is not logically sound.

(iii) You cannot identify any premise as false, but neither can you identify them all as true. In this case, you should explain the situation and apply the definition for the result that you are not in a position to say whether the argument is logically sound.

So given a valid argument, there remains a substantive questions about soundness. In some cases, as for example (ZZ ) on p. ??, this can be the most controversial part.

Again, given an argument, we say in step (a) what would be the case in any story that makes the premises true and the conclusion false. Then, at step (b), instead of finding a consistent story in which the premises are true and conclusion false, we show that there is no such thing. Steps (c) and (d) apply the definitions for the final results. Observe that only one method can be correctly applied in a given case! If we can produce a consistent story according to which the premises are true and the conclusion is false, then it is not the case that no consistent story makes the premises true and the conclusion false. Similarly, if no consistent story makes the premises true and the conclusion false, then we will not be able to produce a consistent story that makes the premises true and the conclusion false.

For showing validity, the most difficult steps are (a) and (b), where we say what happens in every story where the premises true and the conclusion false. For an example, consider the following argument.

\[
\begin{align*}
\text{All collies can fly} \\
\text{All collies are dogs} \\
\text{All dogs can fly}
\end{align*}
\]  
\(G\)

It is invalid. We can easily tell a story that makes the premises true and the conclusion false — say one where collies fly but dachshunds do not. Suppose, however, that we proceed with the validity test as follows,

a. In any story with the premises true and conclusion false,
1. All collies can fly
2. All collies are dogs
3. No dogs can fly

b. In any such story,
   Given (1) and (2),
   4. Some dogs can fly
   Given (3) and (4),
   5. Some dogs can and cannot fly

c. So no consistent story makes the premises true and conclusion false; so by
definition, the argument is logically valid.

d. Since in the real world collies cannot fly, the first premise is not true. So, though
   the argument is logically valid, by definition it is not logically sound.

The reasoning at (b), (c) and (d) is correct. Any story with (1) - (3) is inconsistent.
But something is wrong. (Can you see what?) There is a mistake at (a): It is not
the case that every story that makes the premises true and conclusion false includes
(3). The negation of “All dogs can fly” is not “No dogs can fly,” but rather, “Not all
dogs can fly” (or “Some dogs cannot fly”). All it takes to falsify the claim that all
dogs fly is some dog that does not. Thus, for example, all it takes to falsify the claim
that everyone will get an ‘A’ is one person who does not (on this, see the extended
discussion on p. 28). So for argument (G) we have indeed shown that every story
of a certain sort is inconsistent, but have not shown that every story which makes the
premises true and conclusion false is inconsistent. In fact, as we have seen, there are
consistent stories that make the premises true and conclusion false.

Similarly, in step (b) it is easy to get confused if you consider too much information
at once. Ordinarily, if you focus on sentences singly or in pairs, it will be clear what
must be the case in every story including those sentences. It does not matter which
sentences you consider in what order, so long as in the end, you reach a contradiction
according to which something is and is not so.

So far, we have seen our procedures applied in contexts where it is given ahead
of time whether an argument is valid or invalid. But not all situations are so simple.
In the ordinary case, it is not given whether an argument is valid or invalid. In this
case, there is no magic way to say ahead of time which of our two tests, IT or VT
applies. The only thing to do is to try one way — if it works, fine. If it does not, try
the other. It is perhaps most natural to begin by looking for stories to pry the premises
Chapter 1. Logical Validity and Soundness

Negation and Quantity

In general you want to be careful about negations. To negate any claim \( P \) it is always correct to write simply, it is not the case that \( P \). You may choose to do this for conclusions in the first step of our procedures. At some stage, however, you will need to understand what the negation comes to. We have chosen to offer interpreted versions in the text. It is easy enough to see that,

\[
\text{My mother is a car} \quad \text{and} \quad \text{My mother is not a car}
\]
	negate one another. However, there are cases where caution is required. This is particularly the case with terms involving quantities.

Say the conclusion of your argument is, ‘there are at least ten apples in the basket’. Clearly a story according to which there are, say, three apples in the basket makes this conclusion false. However, there are other ways to make the conclusion false — as if there are two apples or seven. Any of these are fine for showing invalidity.

But when you show that an argument is valid, you must show that any story that makes the premises true and conclusion false is inconsistent. So it is not sufficient to show that stories with (the premises true and) three apples in the basket contradict. Rather, you need to show that any story that includes the premises and less than ten apples fails. Thus in step (a) of our procedure we always say what is so in every story that makes the premises true and conclusion false. So, in (a) you would have the premises and, ‘there are less than ten apples in the basket’.

If a statement is included in some range of consistent stories, then its negation says what is so in all the others — all the ones where it is not so.

That is why the negation of ‘there are at least ten’ is ‘there are less than ten’.

The same point applies with other quantities. Consider some grade examples: First, if a professor says, “everyone will not get an ‘A’,” she says something disastrous — nobody in your class will get an ‘A’. In order too deny it, to show that she is wrong, all you need is at least one person that gets an ‘A’. In contrast, if she says, “someone will not get an ‘A’,” she says only what you expect from the start — that not everyone will get an ‘A’. To deny this, you would need that everyone gets an ‘A’. Thus the following pairs negate one another.

\[
\begin{align*}
\text{Everyone will not get an ‘A’} & \quad \text{and} \quad \text{Someone will get an ‘A’} \\
\text{Someone will not get an ‘A’} & \quad \text{and} \quad \text{Everyone will get an ‘A’}
\end{align*}
\]

It is difficult to give rules to cover all the cases. The best is just to think about what you are saying, perhaps with reference to examples like these.
off the conclusion. If you can find a consistent story to make the premises true and conclusion false, the argument is invalid. If you cannot find any such story, you may begin to suspect that the argument is valid. This suspicion does not itself amount to a demonstration of validity! But you might try to turn your suspicion into such a demonstration by attempting the validity method. Again, if one procedure works, the other better not!

E1.4. Use our validity procedure to show that each of the following is logically valid, and decide (if you can) whether it is logically sound.

*a. If Bill is president, then Hillary is first lady
   Hillary is not first lady
   ______
   Bill is not president

b. Only fools find love
   Elvis was no fool
   ______
   Elvis did not find love

c. If there is a good and omnipotent god, then there is no evil
   There is evil
   ______
   There is no good and omnipotent god

d. All sparrows are birds
   All birds fly
   ______
   All sparrows fly

e. All citizens can vote
   Hannah is a citizen
   ______
   Hannah can vote

E1.5. Use our procedures to say whether the following are logically valid or invalid, and sound or unsound. Hint: You may have to do some experimenting to decide whether the arguments are logically valid or invalid — and so decide which procedure applies.

a. If Bill is president, then Hillary is first lady
   Bill is president
   ______
   Hillary is first lady
b. Most professors are insane
   TR is a professor
   \[\text{T}R \text{ is insane}\]

*c. Some dogs have red hair
   Some dogs have long hair
   \[\text{Some dogs have long, red hair}\]

d. If you do not strike the match, then it does not light
   The match lights
   \[\text{You strike the match}\]

e. Shaq is taller than Kobe
   Kobe is at least as tall as TR
   \[\text{Kobe is taller than TR}\]

1.3 Some Consequences

We now know what logical validity and soundness are, and should be able to identify them in simple cases. Still, it is one thing to know what validity and soundness are, and another to know why they matter. So in this section I turn to some consequences of the definitions.

1.3.1 Soundness and Truth

First, a consequence we want: The conclusion of every sound argument is true in the real world. Observe that this is not part of what we require to show that an argument is sound. LS requires just that an argument is valid and that its premises are true. However it is a consequence of validity plus true premises that the conclusion is true as well.

\[
\text{sound} \implies \text{valid + true premises} \implies \text{true conclusion}
\]

To see this, consider a two-premise argument. Say the real story describes the real world; so the sentences of the real story are all true in the real world. Then in the real story, the premises and conclusion of our argument must fall into one of the following combinations of true and false:
These are all the combinations of T and F. Say the argument is logically valid; then no consistent story makes the premises true and the conclusion false. But the real story is a consistent story. So we can be sure that the real story does not result in combination (2). So far, the real story might result in any of the other combinations. Thus the conclusion of a valid argument may or may not be true in the real world. Now say the argument is sound; then it is valid and all its premises are true in the real world. Again, since it is valid, the real story does not result in combination (2). And since the premises of a sound argument are true in the real world, we can be sure that the premises do not fall into any of the combinations (3) - (8). (1) is the only combination left: in the real story, and so in the real world, the conclusion of a sound argument is true. And not only in this case but in general, if an argument is sound its conclusion is true in the real world: Since a sound argument is valid, there is no consistent story where its premises are true and the conclusion is false, and since the premises really are true, the conclusion has to be true as well. Put another way, if an argument is sound, its premises are true in the real story; but then if the conclusion is false, the real story has the premises true and conclusion false — and the argument is not valid. So if an argument is sound, if it is valid and its premises are true, its conclusion must be true.

Note again: we do not need that the conclusion is true in the real world in order to decide that an argument is sound; saying that the conclusion is true is no part of our procedure for validity or soundness! Rather, by discovering that an argument is logically valid and that its premises are true, we establish that it is sound; this gives us the result that its conclusion therefore is true. And that is just what we want.

1.3.2 Validity and Form

It is worth observing a connection between what we have done and argument form. Some of the arguments we have seen so far are of the same general form. Thus both of the arguments on the left have the form on the right.

If Joe works hard, then he will get an ‘A’

Joe works hard

Joe will get an ‘A’

If Hannah is a citizen then she can vote

Hannah is a citizen

Hannah can vote

If P then Q

P

Q
As it turns out, all arguments of this form are valid. In contrast, the following arguments with the indicated form are not.

<table>
<thead>
<tr>
<th>If Joe works hard then he will get an ‘A’</th>
<th>If Hannah can vote, then she is a citizen</th>
<th>If ( P ) then ( Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joe will get an ‘A’</td>
<td>Hannah is a citizen</td>
<td>( Q )</td>
</tr>
<tr>
<td>Joe works hard</td>
<td>Hannah can vote</td>
<td>( P )</td>
</tr>
</tbody>
</table>

There are stories where, say, Joe cheats for the ‘A’, or Hannah is a citizen but not old enough to vote. In these cases, there is some way to obtain condition \( Q \) other than by having \( P \) — this is what the stories bring out. And, generally, it is often possible to characterize arguments by their forms, where a form is valid iff every instance of it is logically valid. Thus the first form listed above is valid, and the second not.

In chapters to come, we take advantage of certain very general formal or structural features of arguments to identify ones that are valid and ones that are invalid. For now, though, it is worth noting that some presentations of critical reasoning (which you may or may not have encountered), take advantage of patterns like those above, listing typical ones that are valid, and typical ones that are not (for example, Cederblom and Paulsen, Critical Reasoning). A student may then identify valid and invalid arguments insofar as they match the listed forms. This approach has the advantage of simplicity — and one may go quickly to applications of the logical notions for concrete cases. But the approach is limited to application of listed forms, and so to a very narrow range of arguments. In contrast, our approach based on definition LV has application to arbitrary arguments. Further, a mere listing of valid forms does not explain their relation to truth, where the definition is directly connected. Finally, for our logical machine, within a certain range we shall develop an account of validity for quite arbitrary forms. So we are pursuing a general account or theory of validity that goes well beyond the mere lists of these other more traditional approaches.  

1.3.3 Relevance

Another consequence seems less welcome. Consider the following argument.

---

4Some authors introduce a notion of formal validity (maybe in the place of logical validity as above) such that an argument is formally valid iff it has some valid form. As above, formal validity is parasitic on logical validity, together with a to-be-specified notion of form. But if an argument is formally valid, it is logically valid. So if our logical machine is adequate to identify formal validity, it identifies logical validity as well.
It is natural to think that the premises are not connected to the conclusion in the right way — for the premises have nothing to do with the conclusion — and that this argument therefore should not be logically valid. But if it is not valid, by definition, there is a consistent story that makes the premises true and the conclusion false. And, in this case, there is no such story, for no consistent story makes the premises true. Thus, by definition, this argument is logically valid. The procedure applies in a straightforward way. Thus,

a. In any story that makes the premises true and conclusion false,
   1. Snow is white
   2. Snow is not white
   3. Some dogs cannot fly

b. In any such story,
   Given (1) and (2),
   4. Snow is and is not white

c. So no consistent story makes the premises true and conclusion false; so by definition, the argument is logically valid.

d. Since in the real world snow is white, the second premise is not true. So, though the argument is logically valid, by definition it is not logically sound.

This seems bad! Intuitively, there is something wrong with the argument. But, on our official definition, it is logically valid. One might rest content with the observation that, even though the argument is logically valid, it is not logically sound. But this does not remove the general worry. For this argument,

(K)
   There are fish in the sea
   Nothing is round and not round

has all the problems of the other and is logically sound as well. (Why?) One might, on the basis of examples of this sort, decide to reject the (classical) account of validity.
with which we have been working. Some do just this.\footnote{Especially the so-called “relevance” logicians. For an introduction, see Graham Priest, \textit{Non-Classical Logics}. But his text presumes mastery of material corresponding to ?? and ?? (or at least ?? with ??) of this one. So the non-classical approaches develop or build on the classical one developed here.} But, for now, let us see what can be said in defense of the classical approach. (And the classical approach is, no doubt, the approach you have seen or will see in any standard course on critical thinking or logic.)

As a first line of defense, one might observe that the conclusion of every sound argument is true and ask, “What more do you want?” We use arguments to demonstrate the truth of conclusions. And nothing we have said suggests that sound arguments do not have true conclusions: An argument whose premises are inconsistent is sure to be unsound. And an argument whose conclusion cannot be false is sure to have a true conclusion. So soundness may seem sufficient for our purposes. Even though we accept that there remains something about argument goodness that soundness leaves behind, we can insist that soundness is useful as an intellectual tool. Whenever it is the truth or falsity of a conclusion that matters, we can profitably employ the classical notions.

But one might go further, and dispute even the suggestion that there is something about argument goodness that soundness leaves behind. Consider the following two argument forms.

\[
\begin{array}{c}
\text{(ds) } \quad \frac{P \text{ or } Q, \not P}{Q} \\
\text{(add) } \quad \frac{P}{P \text{ or } Q}
\end{array}
\]

According to \textit{ds} (disjunctive syllogism), if you are given that \(P \text{ or } Q\) and that \(\not P\), you can conclude that \(Q\). If you have cake or ice cream, and you do not have cake, you have ice cream; if you are in California or New York, and you are not in California, you are in New York; and so forth. Thus \(\text{ds}\) seems hard to deny. And similarly for \textit{add} (addition). Where ‘or’ means ‘one or the other or both’, when you are given that \(P\), you can be sure that \(P \text{ or } \text{anything}\). Say you have cake, then you have cake or ice cream, cake or brussels sprouts, and so forth; if grass is green, then grass is green or pigs have wings, grass is green or dogs fly, and so forth.

Return now to our problematic argument. As we have seen, it is valid according to the classical definition \textit{LV}. We get a similar result when we apply the \textit{ds} and \textit{add} principles.
If snow is white, then snow is white or anything. So snow is white or dogs fly. So we use line 1 with add to get line 3. But if snow is white or dogs fly, and snow is not white, then dogs fly. So we use lines 2 and 3 with ds to reach the final result. So our principles ds and add go hand-in-hand with the classical definition of validity. The argument is valid on the classical account; and with these principles, we can move from the premises to the conclusion. If we want to reject the validity of this argument, we will have to reject not only the classical notion of validity, but also one of our principles ds or add. And it is not obvious that one of the principles should go. If we decide to retain both ds and add then, seemingly, the classical definition of validity should stay as well. If we have intuitions according to which ds and add should stay, and also that the definition of validity should go, we have conflicting intuitions. Thus our intuitions might, at least, be sensibly resolved in the classical direction.

These issues are complex, and a subject for further discussion. For now, it is enough for us to treat the classical approach as a useful tool: It is useful in contexts where what we care about is whether conclusions are true. And alternate approaches to validity typically develop or modify the classical approach. So it is natural to begin where we are, with the classical account. At any rate, this discussion constitutes a sort of acid test: If you understand the validity of the “snow is white” and “fish in the sea” arguments (J) and (K), you are doing well — you understand how the definition of validity works, with its results that may or may not now seem controversial. If you do not see what is going on in those cases, then you have not yet understood how the definitions work and should return to section 1.2 with these cases in mind.

E1.6. Use our procedures to say whether the following are logically valid or invalid, and sound or unsound. Hint: You may have to do some experimenting to decide whether the arguments are logically valid or invalid — and so decide which procedure applies.

a. Bob is over six feet tall
   Bob is under six feet tall
   Bob is disfigured
b. Marilyn is not over six feet tall
   Marilyn is not under six feet tall
   Marilyn is not in the WNBA

c. There are fish in the sea
   Nothing is round and not round

d. Cheerios are square
   Chex are round
   There is no round square

e. All dogs can fly
   Fido is a dog
   Fido cannot fly
   I am blessed

E1.7. Respond to each of the following.

a. Create another argument of the same form as the first set of examples (H) from section 1.3.2, and then use our regular procedures to decide whether it is logically valid and sound. Is the result what you expect? Explain.

b. Create another argument of the same form as the second set of examples (I) from section 1.3.2, and then use our regular procedures to decide whether it is logically valid and sound. Is the result what you expect? Explain.

E1.8. Which of the following are true, and which are false? In each case, explain your answers, with reference to the relevant definitions. The first is worked as an example.

a. A logically valid argument is always logically sound.
   False. An argument is sound iff it is logically valid and all of its premises are true in the real world. Thus an argument might be valid but fail to be sound if one or more of its premises is false in the real world.

b. A logically sound argument is always logically valid.

*c. If the conclusion of an argument is true in the real world, then the argument must be logically valid.
d. If the premises and conclusion of an argument are true in the real world, then the argument must be logically sound.

*e. If a premise of an argument is false in the real world, then the argument cannot be logically valid.

f. If an argument is logically valid, then its conclusion is true in the real world.

*g. If an argument is logically sound, then its conclusion is true in the real world.

h. If an argument has contradictory premises (its premises are true in no consistent story), then it cannot be logically valid.

*i. If the conclusion of an argument cannot be false (is false in no consistent story), then the argument is logically valid.

j. The premises of every logically valid argument are relevant to its conclusion.

E1.9. For each of the following concepts, explain in an essay of about two pages, so that (high-school age) Hannah could understand. In your essay, you should:
(i) identify the objects to which the concept applies, (ii) give and explain the definition, and give and explicate examples of your own construction (iii) where the concept applies, and (iv) where it does not. Your essay should exhibit an understanding of methods from the text.

a. Logical validity

b. Logical soundness

E1.10. Do you think we should accept the classical account of validity? In an essay of about two pages, explain your position, with special reference to difficulties raised in section 1.3.3.

Answers to Selected Exercises

1.4 Chapter One

E1.1. Say whether each of the following stories is internally consistent or inconsistent. In either case, explain why.

Exercise 1.1
ANSWERS FOR CHAPTER 1

a. Smoking cigarettes greatly increases the risk of lung cancer, although most people who smoke cigarettes do not get lung cancer.

*Consistent.* Even though the risk of cancer goes up with smoking, it may be that most people who smoke do not have cancer. Perhaps 49% of people who smoke get cancer, and 1% of people who do not smoke get cancer. Then smoking greatly increases the risk, even though most people who smoke do not get it.

c. Abortion is always morally wrong, though abortion is morally right in order to save a woman’s life.

*Inconsistent.* Suppose (whether you agree or not) that abortion is *always* morally wrong. Then abortion is wrong even in the case when it would save a woman’s life. So the story requires that abortion is and is not wrong.

e. No rabbits are nearsighted, though some rabbits wear glasses.

*Consistent.* One reason for wearing glasses is to correct nearsightedness. But glasses may be worn for other reasons. It might be that rabbits who wear glasses are farsighted, or have astigmatism, or think that glasses are stylish. Or maybe they wear sunglasses just to look cool.


g. Barack Obama was never president of the United States, although Michelle is president right now.

*Consistent.* Do not get confused by the facts! In a story it may be that Barack was never president and his wife was. Thus this story does not contradict itself and is consistent.

i. The death star is a weapon more powerful than that in any galaxy, though there is, in a galaxy far, far away, a weapon more powerful than it.

*Inconsistent.* If the death star is more powerful than any weapon in any galaxy, then according to this story it is and is not more powerful than the weapon in the galaxy far far away.

E1.2. For each of the following sentences, (i) say whether it is true or false in the real world and then (ii) say, if you can, whether it is true or false according to the accompanying story. In each case, explain your answers.

c. Sentence: After overrunning Phoenix in early 2006, a herd of buffalo overran Newark, New Jersey.


*Exercise 1.2.c*
(i) It is *false* in the real world that any herd of buffalo overran Newark anytime after 2006. (ii) And, though the story says something about Phoenix, the story does not contain enough information to say whether the sentence regarding Newark is true or false.

e. Sentence: Jack Nicholson has swum the Atlantic.

   Story: No human being has swum the Atlantic. Jack Nicholson and Bill Clinton and you are all human beings, and at least one of you swam all the way across!

   (i) It is *false* in the real world that Jack Nicholson has swum the Atlantic. (ii) This story is inconsistent! It requires that some human both has and has not swum the Atlantic. Thus we refuse to say that it makes the sentence true or false.

g. Sentence: Your instructor is not a human being.

   Story: No beings from other planets have ever made it to this country. However, your instructor made it to this country from another planet.

   (i) Presumably, the claim that your instructor is not a human being is *false* in the real world (assuming that you are not working by independent, or computer-aided study). (ii) But this story is inconsistent! It says both that no beings from other planets have made it to this country and that some being has. Thus we refuse to say that it makes any sentence true or false.

i. Sentence: The Yugo is the most expensive car in the world.

   Story: Jaguar and Rolls Royce are expensive cars. But the Yugo is more expensive than either of them.

   (i) The Yugo is a famously cheap automobile. So the sentence is *false* in the real world. (ii) According to the story, the Yugo is more expensive than some expensive cars. But this is not enough information to say whether it is the most expensive car in the world. So there is not enough information to say whether the sentence is true or false.

E1.3. Use our invalidity test to show that each of the following arguments is not logically valid, and so not logically sound.

*For each of these problems, different stories might do the job.

a. If Joe works hard, then he will get an ‘A’

   *Joe will get an ‘A’*

   *Joe works hard*

   *Exercise 1.3.a*
a. In any story with premises true and conclusion false,
   1. If Joe works hard, then he will get an ‘A’
   2. Joe will get an ‘A’
   3. Joe does not work hard
b. Story: Joe is very smart, and if he works hard, then he will get an ‘A’. Joe will get an ‘A’; however, Joe cheats and gets the ‘A’ without working hard.
c. This is a consistent story that makes the premises true and the conclusion false; thus, by definition, the argument is not logically valid.
d. Since the argument is not logically valid, by definition, it is not logically sound.

E1.4. Use our validity procedure to show that each of the following is logically valid, and decide (if you can) whether it is logically sound.

*For each of these problems, particular reasonings might take different forms.

a. If Bill is president, then Hillary is first lady
   Hillary is not first lady
   Bill is not president

   a. In any story with premises true and conclusion false,
      1. If Bill is president, then Hillary is first lady
      2. Hillary is not first lady
      3. Bill is president

   b. In any such story,
      Given (1) and (3),
      4. Hillary is first lady
      Given (2) and (4),
      5. Hillary is and is not first lady

   c. So no story with the premises true and conclusion false is a consistent story; so by definition, the argument is logically valid.

   d. In the real world Hillary is not first lady and Bill and Hillary are married so it is true that if Bill is president, then Hillary is first lady; so all the premises are true and by definition the argument is logically sound.

   Exercise 1.4.a
E1.5. Use our procedures to say whether the following are logically valid or invalid, and sound or unsound. Hint: You may have to do some experimenting to decide whether the arguments are logically valid or invalid — and so decide which procedure applies.

c. Some dogs have red hair
   Some dogs have long hair
   ______
   Some dogs have long, red hair

   a. In any story with the premise true and conclusion false,
      1. Some dogs have red hair
      2. Some dogs have long hair
      3. No dogs have long, red hair

   b. Story: There are dogs with red hair, and there are dogs with long hair. However, due to a genetic defect, no dogs have long, red hair.
   c. This is a consistent story that makes the premise true and the conclusion false; thus, by definition, the argument is not logically valid.
   d. Since the argument is not logically valid, by definition, it is not logically sound.

E1.6. Use our procedures to say whether the following are logically valid or invalid, and sound or unsound.

d. Cheerios are square
   Chex are round
   ______
   There is no round square

   a. In any story with the premises true and conclusion false,
      1. Cheerios are square
      2. Chex are round
      3. There is a round square

   b. In any such story, given (3),
      4. Something is round and not round

   c. So no story with the premises true and conclusion false is a consistent story; so by definition, the argument is logically valid.

*Exercise 1.6.d*
1.8

d. In the real world Cheerios are not square and Chex are not round, so the premises are not true; so though the argument is valid, by definition it is not logically sound.

E1.8. Which of the following are true, and which are false? In each case, explain your answers, with reference to the relevant definitions.

c. If the conclusion of an argument is true in the real world, then the argument must be logically valid.

*False.* An argument is logically valid iff there is no consistent story that makes the premises true and the conclusion false. Though the conclusion is true in the real world (and so in the real story), there may be some other story that makes the premises true and the conclusion false. If this is so, then the argument is not logically valid.

e. If a premise of an argument is false in the real world, then the argument cannot be logically valid.

*False.* An argument is logically valid iff there is no consistent story that makes the premises true and the conclusion false. For logical validity, there is no requirement that every story have true premises — only that ones that do, also have true conclusions. So an argument might be logically valid, and have premises that are false in many stories, including the real story.

g. If an argument is logically sound, then its conclusion is true in the real world.

*True.* An argument is logically sound iff it is logically valid and its premises are true in the real world. Since the premises are true in the real world, they hold in the real story; since the argument is valid, this story cannot be one where the conclusion is false. So the conclusion of a sound argument is true in the real world.

i. If the conclusion of an argument cannot be false (is false in no consistent story), then the argument is logically valid.

*True.* If there is no consistent story where the conclusion is false, then there is no consistent story where the premises are true and the conclusion is false; but an argument is logically valid iff there is no consistent story where the premises are true and the conclusion is false. So the argument is logically valid.
Bibliography


