**Probability Primer**

**Basic Principles**

1. \( P(A \text{ or not-}A) = 1 \)
   logical truths always have probability 1

2. \( P(A \text{ and not-}A) = 0 \)
   contradictions always have probability 0

3. If \( A \) is logically equivalent to \( B \), then \( P(A) = P(B) \)

4. \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \)
   Consequence: \( P(\text{not-}A) = 1 - P(A) \)

5. \( P(A \text{ and } B) = P(A) \times P(B \mid A) \)
   \( P(B \mid A) \) is \( P(B) \) given that \( A \)

**Results**

6. Bayes’s Theorem
   \[
   P(B \mid A) = \frac{P(B) \times P(A \mid B)}{P(A)} = \frac{P(B) \times P(A \mid B)}{P(B) \times P(A \mid B) + P(\text{not-}B) \times P(A \mid \text{not-}B)}
   \]

7. Bayes’s Theorem (again)
   \[
   P(E \mid tE) = \frac{P(E) \times P(tE \mid E)}{P(E) \times P(tE \mid E) + P(\sim E) \times P(tE \mid \sim E)} = \frac{1}{1 + \frac{P(\sim E) \times P(tE \mid \sim E)}{P(E) \times P(tE \mid E)}}
   \]

8. A ratio from Bayes’s Theorem
   \[
   \frac{P(E \mid tE)}{P(\sim E \mid tE)} = \frac{P(tE \mid E)}{P(tE \mid \sim E)} \times \frac{P(E)}{P(\sim E)}
   \]

9. Independence
   Say \( P(t_2E \mid t_1E \& E) = P(t_2E \mid E) \); then \( P(t_1E \& t_2E \mid E) = P(t_1E \mid E) \times P(t_2E \mid E) \)